AN ANALYSIS OF THE *MANDAPHALA* TABLES OF MAKARANDA AND REVISION OF PARAMETERS

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ABSTRACT

Among the Indian astronomical tables the most popular one is the text called *Makaranda Sāriṇī* (*MKS*, *Sāriṇī* means tables) composed by the medieval astronomer Makaranda (c. 1478 C.E.). The equation of the centre (*mandaphala*) for the five planets is treated differently from the traditional procedure. In the present paper we analyze mathematically the formation of these *mandaphala* tables of Makaranda. We compare the *mandaphala* values of *MKS* with those of *Sūryasiddhānta* (*SS*) and the departure in the values is marked. The critical *manda* anomaly (*mandakendra*) for each planet is obtained correct to an integer from the *MKS* tables and more precisely by considering the analytical expression. These critical values and the related maximum equation of the centre (*mandaphala*) are compared with those obtained by the modern procedure. Further there is a great need to revise and update the parameters and procedures for compilation of the annual Indian astronomical almanacs (*pañcāṅgas*). We have suggested a few changes by which the values of the *mandaphalas* of the Sun, the Moon and the planets are closer to the modern scientific values.

Keywords: Makaranda, *sāriṇī, astronomical tables, mandaphala, pañcāṅga.*

I. INTRODUCTION

Compilation and use of annual *pañcāṅgas* is a socio-religious necessity in the Hindu society. These almanacs are based on traditional astronomical treatises (*siddhānta*) like the *Sūryasiddhānta* (*SS*), *Brahmasiddhānta* etc. For actual computations of the almanacs generally astronomical tables are used, since the direct application of the major texts is cumbersome and tedious for day to day positions of the heavenly bodies.

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The popularly used Indian astronomical tables belong to different schools (*pakṣas*) based on the different major texts. The major schools are (i) the *Saurapakṣa*, (ii) the *Āryapakṣa*, (iii) the *Brāmhapakṣa* and (iv) the *Gaṇeśapakṣa*. These different *pakṣas* conformed to the parameters and procedures respectively of the *Sūryasiddhānta*, *Āryabhaṭīyam* of Āryabhaṭa I (born 476 C.E.), *Brahma Sphuṭasiddhānta* of Brahmagupta (c. 628 C.E.) and the *Grahalāghavam*(*GL*) of Gaṇeśa Daivajña (c. 1520 C.E.).

The Indian astronomical tables are called differently as *Sāriṇī*, *padakams*, *koṣṭhakas* and *vākyas*. The major tables of the *saurapakṣa* are (i) *Makarandasāriṇī*, (ii) *Gaṇakānanda*, (iii) *Pratibhāgī* and (iv) *Tyāgarti* manuscripts. Among these *Makarandasāriṇī* of Makaranda is the most popular one. The *Gaṇakānanda* of Sūrya is a *karaṇa* text (handbook) popular mainly in Andhra Pradesh and Karnataka. *Pratibhāgī* and *Tyāgarti* tables are used among the *saurapakṣa* followers in Karnataka. Similarly there are tables belonging to the other schools also. In fact, the *vākya* tables used mainly in Kerala and Tamilnadu composed by a legendary person called Vararuci comprise simple Sanskrit sentences which are numerical chronograms based on the *Kaṭapayādi* system.

2. TRUE POSITIONS OF PLANETS ACCORDING TO *SS*

In Indian classical procedures for the true positions of heavenly bodies two important equations (*saṃskāras*) are applied to the mean (*madhyama*) positions. These are the *mandaphala* and the *śīghraphala*. The first one is the equation of the centre which is applied since a heavenly body's orbit is eccentric and hence its motion is not uniform. The second correction *śīghraphala* (equation of the conjunction) corresponds to transforming the heliocentric true position to the geocentric true position of the five planets (*tārā-grahas*) viz., Mars (Kuja), Mercury (Budha), Jupiter (Guru), Venus (Śukra) and Saturn (Śani). However, in the case of the Sun and the Moon only one correction viz. the equation of the centre is applied.

According to the *Sūryasiddhānta* (*SS*) the procedure of obtaining the true position of a planet (*tārā-graha*) consists of the following operations.

1st Operation

Suppose *MP* is the mean position of a planet. Its *śīghra* anomaly (*kendra*) is defined as *śīghrakendra, SK* =*śīghrocca* - *MP*. Here *śīghrocca* is the apex of *SK* and taken as the mean Sun for the superior planets viz., Kuja, Guru and Śani. On the other hand, in the case of the inferior planets Budha and Śukra the mean Sun is itself taken as the mean planet. The *śīghrocca* is the actual mean position of Budha or Śukra as the case may be.

For the *SK* thus defined the corresponding *śīghra* equation *SE* is determined for this first operation. In this case let us denote *SK* and *SE* as SK_1 and SE_1 respectively. Now half of SE_1 is applied to *MP* and thus the first *śīghra* corrected planet P_1 is obtained as shown in (2.1):

i.e. $P_1 = MP + \frac{1}{2}(SE_1)$. ...(2.1)

2nd Operation

To the thus obtained first *śīghra* corrected planet from the 1st operation, half *mandaphala* is applied. For this, the *manda* anomaly (*kendra*) *MK* is defined as

$MK = mandocca - P_1$.

Here, the *mandocca* is the apogee (or aphelion). While the *mandocca* of the Moon moves at a mean rate of about 6'41'' per day, the *mandoccas* of the Sun and the planets are taken as fixed in most of the Indian classical texts. For example the sidereal longitude of the Sun's *mandocca* is taken as 78°. However, some texts like the *Sūryasiddhānta* and Nīlakaṇṭha Somayājī's *Siddhānta darpaṇa* prescribe a certain small rate of motion for each of these bodies in terms of revolutions per *kalpa* (432 x 10^7 years). That these texts speculate motion for the *mandoccas* of the Sun and the planets is to be appreciated. The values given for the rates of motion are however unrealistic.

For the *MK* thus defined the corresponding equation (*mandaphala*, *MPH*)is determined using the expression

$$\sin(MPH) = \frac{p}{R}\sin\left(MK\right) \qquad ...(2.2)$$

where *p* is the periphery of the *manda* epicycle and *R* (=360°) is that of the deferent circle. These are denoted respectively by MK_1 and MPH_1 for this first *manda* correction. Then the first *manda* corrected planet P_2 is obtained by (2.3):

$$P_2 = P_1 + \frac{1}{2}\left(MPH_1\right) \qquad ...(2.3)$$

3rd Operation

From the position thus corrected we find again the *manda* correction MPH_2 and apply it entirely to the original mean position MP of the planet. Thus the second *manda* corrected planet P_3 is given by (2.4):

$$P_3 = MP + (MPH_2) \qquad \ldots(2.4)$$

4th Operation

From the planet's position obtained from the 3rd operation we find the *śīghra* correction for the second time and apply the *whole* of it to the above. This means that the *śīghra* correction SE_2 obtained from the position P_3 of the planet is applied entirely to P_3. Thus the second *śīghra* corrected planet P_4 is given by (2.5):

$$P_4 = P_3 + (SE_2) \qquad \ldots(2.5)$$

where the second *śīghrakendra*

$SK_2 = \acute{S}\bar{\imath}ghrocca - P_3$.

This four times corrected P_4 is the *true* longitude of the planet.

3. MAKARANDA'S PROCEDURE FOR TRUE PLANETS

Makaranda's procedure for determining the equation of the centre of a planet comprises the following steps. While *SS* provides four operations to get the true position of a planet, *MKS* simplifies the procedure by reducing it to three operations. Instead of the second and third operations, in the *SS* procedure, of applying half *manda* and full *mandasaṃskāras* one after the other, *MKS* applies only once full *manda* correction by combining the two steps. Thus we have the following three successive operations according to *MKS*.

1st Operation

This correction (3.1) is the same as the one (2.1) according to *SS*.

$$P_1 = MP + \frac{1}{2} SE_1 \qquad \ldots(3.1)$$

where the first *śīghrakendra* $SK_1 = \acute{s}\bar{\imath}ghrocca - MP$.

2nd Operation

Here, unlike the *SS* procedure (2.2), in Makaranda's procedure the full *manda* correction is applied as shown in (3.2):

$$P_2 = MP + (MPH_2) \qquad \ldots(3.2)$$

Where *MPH*, is obtained with notation as in (2.2) to (2.4) as follows

$$= \sin^{-1}\left[\frac{p}{R}\sin MK_2\right]$$

$$= \sin^{-1}\left[\frac{p}{R}\sin\left(mandocca - P_2\right)\right]\text{b}$$

$$= \sin^{-1}\left[\frac{p}{R}\sin\left(mandocca - P_1 + \frac{1}{2}MPH_1\right)\right]$$

$$= \sin^{-1}\left[\frac{p}{R}\sin\left(MK_1 + \frac{1}{2}MPH_1\right)\right]$$

which is reflected in (5.3)

3rd Operation

In this last operation, the second *śīghra* correction SE_2 is added fully to P_2 as shown in (3.3):

$$P_3 = P_2 + SE_2). \qquad \ldots(3.3)$$

where the second *śīghrakendra* $SK_2 =$ *Śīghrocca*$-P_2$. Thus the three times corrected P_3 is the true position of a planet according to *MKS*. In the text *SS*, as also many other classical Indian astronomical texts, the *mandaphala* of a heavenly body attains its maximum for the *manda* anomaly (*MK*) equal to 90°. But interestingly in the *manda* tables of *MKS* for the five planets the maximum of each of them is attained for a critical value of *MK* beyond 90°. These critical values of *MK* are different for different planets. However, for the Sun and the Moon the *mandaphala* attains its maxima at *MK*=90°.

Thus we find that Makaranda adopts only three steps, while the classical text has four of them more explicitly.While *SS* applies the *manda* correction twice, *MKS* resorts to the application of the said correction only once with revised values.

4. EQUATION OF THE CENTRE (*MANDAPHALA*) IN *MAKARANDA SĀRIṆĪ* (*MKS*)

For a comparison of the values of the equation of the centre according to *SS* and *MKS* we use result (2.2) for the *manda* anomaly $MK = 0°$ to $180°$ at intervals of 15°. The *manda* periphery (*paridhi*) is a variable given by

$$p = p_e - (p_e - p_o) \; sin \; (MK) \qquad \ldots(4.1)$$

where p_e and p_O are the *manda* peripheries at the ends of the even and odd quadrants. For example in the case of Mars $p_e = 75°$ and $p_o = 72°$.

Thus using results (2.2) and (4.1) we list the values of *mandaphala* (*MPH*) in arc minutes and the same are compared with the corresponding tabular values for *MKS* (p.14-15) in Table 4.1.

Table 4.1: *MPH* of Mars according to *SS* and *MKS* for *MK* varying from 0° to 180°

MK	*Paridhi*(*SS*)	*MPH* (*SS*)	*MPH* (*MKS*)
0°	75	0	0
15°	74.224	183.53	165
30°	73.5	351.55	320
45°	72.879	493.8	458
60°	72.402	601.83	570
75°	72.102	669.28	649
90°	72	692.22	689
105°	72.102	669.28	683
120°	72.401	601.83	629
135°	72.879	493.8	527
150°	73.5	351.55	381
165°	72.224	183.53	202
180°	75	0	0

We notice in Table 4.1 that (i)for $MK = 90°$ the *SS* value for *MPH* is maximum at 692'.22 whereas the *MKS* value is 689', falling short by 3'.22. In fact, *MKS* has listed the value of *MPH* for each degree of *MK* from 0° to 180°. In those tables we find that for each planet the maximum *MPH* is attained not for $MK = 90°$, but for a critical *MK* beyond 90°. Further, the critical *MK* has different value for different planets.

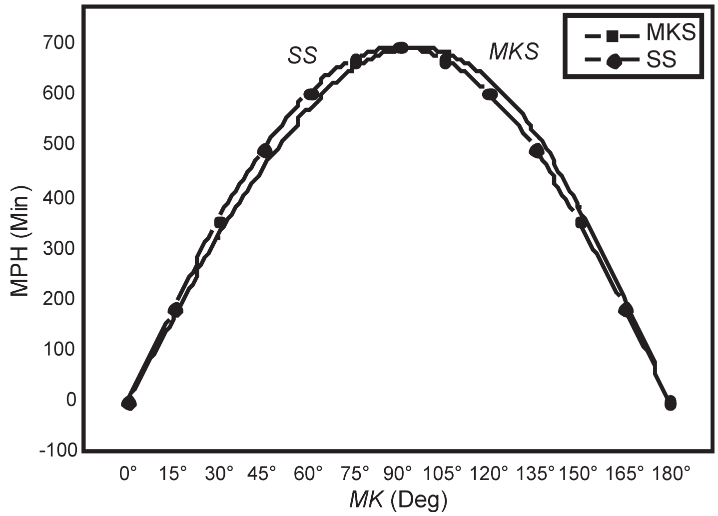

Fig. 4.1

Table 4.2: *MPH* of Mars according to *SS* and *MKS* for *MK* varying from 85° to 100°

MK	*Paridhi*(*SS*)	*MPH* (*SS*)	*MPH* (*MKS*)
85°	72.01141	689.6581	680
86°	72.00731	690.5792	682
87°	72.00411	691.2958	684
88°	72.00183	691.8079	686
89°	72.00046	692.1151	687
90°	72	692.2175	689
91°	72.00046	692.1152	690
92°	72.00183	691.8078	691
93°	72.00411	691.2957	692
94°	72.00731	690.5792	692
95°	72.01141	689.6582	692
96°	72.01643	688.5331	692
97°	72.02236	687.204	692
98°	72.0292	685.6714	692
99°	72.03693	683.9356	691
100°	72.04558	681.9973	691

In order to indicate the variation of *MPH* and its attaining the maximum value we have listed the *MPH*, for Mars, in Table 4.2 for every degree of the *manda* anomaly *MK* from 85° to 100° (p.14-15, *MKS*).

We notice in Table 4.2 that $MPH = 692.2175$ is maximum according to *SS* at the critical $MK = 90°$. On the other hand according to *MKS* maximum $MPH =$ 692 for *MK* ranging from 93° to 98° this wide range for *MK* is due to the reason that the *MPH* values are tabulated in integers, neglecting the fractional part in *MKS*. Similar behavior is seen for other planets also.

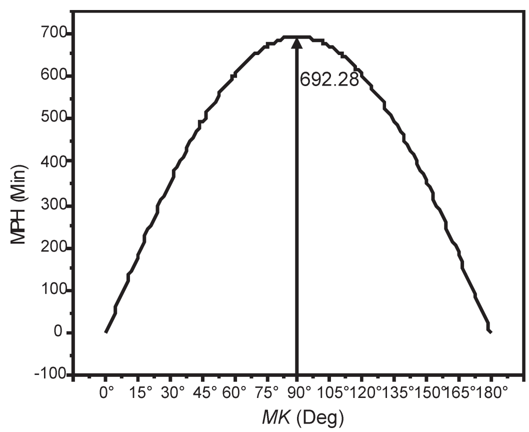

Fig. 2. MPH of Mars according to SS

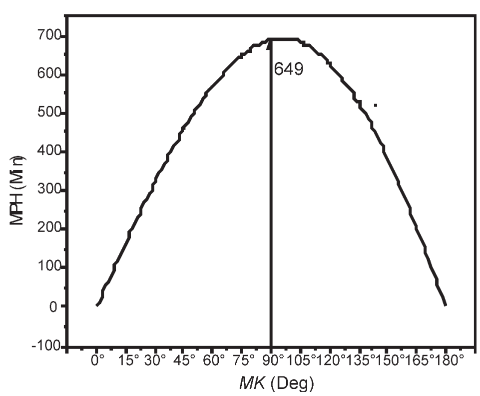

Fig. 3 MPH of Mars according to MKS

The variation of *MPH* against *MK* for different planets according to *SS* and *MKS* are shown graphically in Fig. 4.2 and Fig. 4.3

5. RATIONALE BEHIND THE VALUES OF *MANDAPHALA* OF MAKARANDA

The *MKS* text does not indicate how the value of *MPH* for each degree of the *manda* anomaly *MK* for the five planets is obtained nor do the commentaries explain this. In this section we work out the possible procedure for the same. It consists of the following steps.

Suppose MK_1 denotes, the *manda* anomaly (*mandakendra*) of a planet then the corresponding *manda* periphery (p_1) is given in (5.1), using (4.1) and noting that sin(*MK*) is positive in the first two quadrants:

$$p_1 = p_e - (p_e - p_o) \sin (MK_1) \qquad \ldots(5.1)$$

$$\sin(MPH_1) = \frac{p_1}{R} \sin(MK_1) \qquad \ldots(5.2)$$

Now, we revise the *mandakendra* (MK_1) by adding half of the *mandaphala* obtained from (5.2) and the revised *MPH* is denoted by MPH_2:

$$\sin(MPH_2) = \frac{p_1}{R} \sin(MK_2) \qquad \ldots(5.3)$$

where $MK_2 = MK_1 + \frac{1}{2} MPH_1$. The thus obtained revised values MPH_2 for Mars' original *manda* anomaly MK_1 are listed for MK_1=0° to 180° in Table 5.1

In Table 5.1 we observe that our revised values of *MPH* coincide with the tabulated values of *MKS* (p.14-15) correct to an integer.

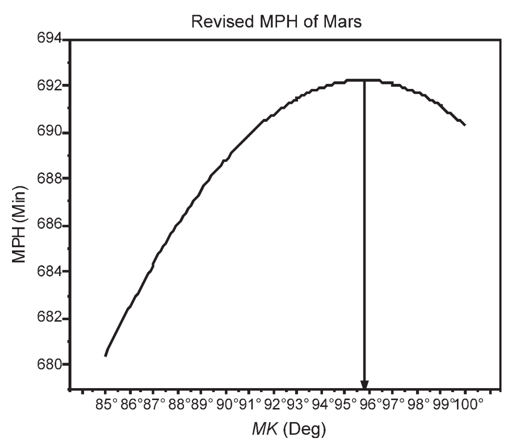

Fig. 5.1

In the case of the other four planets Jupiter, Saturn, Mercury and Venus also the revised values of *MPH* according to our suggested algorithm coincide well with the corresponding tabulated values of *MKS* correct to an integer.

Table 5.1: Revised values MPH_2 for Mars

MK_1	Revised *MPH*	*MPH* (*MKS*)
0°	0	0
15°	165.3442	165
30°	320.4618	320
45°	457.8608	458
60°	569.9568	570
75°	649.2465	649
90°	688.8117	689
105°	683.1411	683
120°	629.1446	629
135°	527.1519	527
150°	381.6481	381
165°	201.5603	202
180°	0	0

Table 5.2: Mars' revised *mandaphala* and *MKS* values for MK 85° to 100°

MK	Revised *MPH*	*MPH* (*MKS*)
85°	680.4169	680
86°	682.4911	682
87°	684.3693	684
88°	686.0497	686
89°	687.531	687
90°	688.8117	689
91°	689.8903	690
92°	690.7659	691
93°	691.4369	692
94°	691.902	692
95°	692.1603	692
96°	692.2105	692
97°	692.0515	692
98°	691.6823	692
99°	691.1018	691
100°	690.3097	691

In order to study the variation of the Mars' *MPH* close to the critical value of *MK* we have computed its values for every degree of *MK* from 85° to 100° (p.14-15, *MKS*).

From Table 5.2 the maximum revised *MPH* for Mars is 692.2105 for critical *MK*=96°. Sharper values for the critical *MK* and the corresponding maximum *MPH* are worked out for different planets and listed, along with the prescribed *manda* periphery p_e and p_o in Table 5.3

Table 5.3: Critical *MK* and Maximum *MPH* for planets

Planet	*Pe*	*Po*	Critical MK Deg.	Maximum MPH *Kalās*
Ravi	14	13.66	91.2	114.9008
Candra	32	31.66	93.9	378.3264
Kuja	75	72	95.74038	692.2175
Budha	30	28	92.22888	697.6506
Guru	33	32	92.54739	305.9813
Śukra	12	11	90.87539	105.0586
Śani	49	48	93.82276	459.7353

Table 5.4: Revised *MPH* for five planets for *MK* = 85° to 100°

MK	*MPH* Kuja	*MPH* Budha	*MPH* Guru	*MPH* Śukra	MPH Śani
85°	680.4169	265.6734	303.4106	104.557	454.3833
86°	682.4911	266.1814	304.0447	104.7131	455.5214
87°	684.3693	266.6145	304.5897	104.8402	456.5251
88°	686.0497	266.9724	305.0453	104.9383	457.3938
89°	687.531	267.255	305.4111	105.0074	458.1267
90°	688.8117	267.4619	305.6871	105.0474	458.7233
91°	689.8903	267.5932	305.6977	105.0584	459.1828
92°	690.7659	267.6486	305.8727	105.0402	459.5046
93°	691.4369	267.628	305.972	104.9929	459.6882
94°	691.902	267.5311	305.8853	104.9164	459.7332
95°	692.1603	267.358	305.7075	104.8107	459.6388
96°	692.2105	267.1085	305.4384	104.6759	459.4048
97°	692.0515	266.7824	305.0778	104.5119	459.0305
98°	691.6823	266.3797	304.6257	104.3188	458.5158
99°	691.1018	265.9004	304.0818	104.0964	457.8605
100°	690.3097	265.3442	303.4463	103.8449	457.0638

6. NEED AND JUSTIFICATION FOR REVISION OF PARAMETERS

Specially in the Indian context, social and religious observances are based on the traditional dating system comprising (i) lunar or a solar month, (ii) the daily *tithi* (one thirtieth) of a lunar month, (iii) the daily *nakṣatra* (one twenty-seventh part of the Moon's sidereal period) etc. Further in Hindu households there is a prevalent practice of casting the horoscope for the time of birth of a child. For this the correct true positions of the planets, besides the Sun and the Moon are required. Currently these computations are made by professional astrologers using the annual traditional almanacs (*pañcāṅgas*). These almanacs are compiled based on the standard texts like *SS*, *GL* etc. Since these texts were composed several centuries ago the positions of the heavenly bodies do not come out correct as compared to the modern scientific computations.

Therefore there is an urgent need to update the relevant parameters as also the mathematical procedures. Towards this aim, one of the wellknown earlier attempts was by Venkatesh Ketkar (c. 1898 C.E.). He published two Sanskrit texts, *Jyotirganitam* (1898 C.E.) and *Ketkayagrahaganitam* (1898 C.E.). In these texts Ketkar used the tables and procedures of Simon New Comb (1835-1909) Leverrier etc. The present day *pañcāṅgas* based on Ketkar's tables refer to themselves as *dṛg-gaṇita*. However, Ketkar's texts are based on the data prevalent during the early second half of the 19th century. Since then substantial changes are effected in the relevant parameters in modern astronomy. Hence the Indian computational parameters have to be suitably revised to obtain reasonably correct calendriceal and planetary data. Then comes the question whether such rather drastic revisions are permitted in the Indian traditional framework of *siddhāntas*. We reproduce below some significant statement and injunctions enjoined in traditional Indian astronomical texts. We refer to a very valuable article by M D Srinivas on this issue. The following are a few excerpts from the said article.

(i) The famous Kerala astronomer Parameswara (1360-1455) states, "In the course of time the corrections (*samskārāḥ*) must be thought over (and implemented) by the best mathematicians.

-kālāntaretusaṃskāraś cintyatāṃgaṇakottamaiaḥ

(ii) In his *Karaṇaratna*, Devācārya (c. 7th century) declares "They say that the aim of acquiring the knowledge of astronomy is to rectify and re-establish the lost methods or to discover and highlight new methods hence this attempt of mine".

(iii) Jyeśṭhadeva (c.1500-1600) in his Malayalam text *Dṛikkaraṇa* describes the long series of revisions performed even with in the *Āryabhaṭīya* school (*Āryapakśa*) of Indian astronomy.

(iv) Commenting on the faint-heartedness of a certain commentator of Manjula's (c. 932 C.E.) *Mānasam* the famous Kerala astronomer Nīlakaṇṭha Somayājī (1444-1545) admonishes,

"—— When *siddhāntas* show discord, observations should be made of revolutions etc. (which would give results which accord with actual observations) and a new *siddhānta* enunciated."

(v) As regards the ploys employed to get finally the correct results commentator Pṛthūdakasvāmi (c. 860 C.E.) of Brahmagupta's *Brahmasputa siddhānta* (628 C.E.) clearly explains, the ploys are temporary and modifiable as follows: "Just as the grammarians employ fictitious entities such as *prakṛti*, *pratyaya*, *āgama*, *lopa*, *vikara* etc. to decide on the established real word forms, and just as the *vaidyās* (medical experts) employ tubes etc. to demonstrate surgery, one has to understand and feel contented that it is in the same way that the astronomers employ notions such as the motion of the planets etc. in *manda* and *śīghrapratimandals* as for the sake of accurate predictions.

With these justifications and the requirement to revise the parameters with in the classical framework in the following section we suggest a few suitable revisions.

7. REVISED *MANDA* PERIPHERIES FOR THE HEAVENLY BODIES

As explained in section 2 the procedure for the *manda* equation (equation of the centre) to be applied to the mean position of the Sun, the Moon and the planets consists of the expression (2.2):

$$\sin(MPH) = \frac{p_1}{R}\sin(MK) \qquad \ldots(7.1)$$

where *MPH* is the equation of the centre (*mandaphala*), *p* is the *manda* periphery in degrees and $R = 360°$ and *MK* is the *manda* anomaly (*mandakendra*).

The corresponding modern expression for the equation of the centre, considering the first two terms, is

$$MPH = \left(2e - \frac{1}{4}e^3\right)\sin m + \left(\frac{5}{4}e^2 - \frac{11}{24}e^4\right)\sin 2m \qquad \ldots(7.2)$$

where e is the eccentricity of the elliptical orbit of the concerned body. Since generally e is small, except for Pluto and Mercury, ignoring the higher powers of e, the equation of the centre can be approximated as

$$MPH = (2e)\sin m \qquad \ldots(7.3)$$

Comparing equations (2.2) and (7.3) we get the *manda* periphery for the Sun as p=12°.0607488 by taking the eccentricity of the earth's orbit as 0.01675104.

However in proposing *bīja* (correction) to the peripheries of the *manda* epicycle of the heavenly bodies we, now consider even the higher powers of e, viz. e^2, e^3, e^4. Considering the first two terms in the sine-series for *MPH* let

$$E_1 = \left(2e - \frac{1}{4}e^3\right) \quad \text{and} \quad E_2 = \left(\frac{5}{4}e^2 - \frac{11}{24}e^4\right)$$

Then (7.2) can be written as $MPH = E_1 \, sin \; m + E_2 \, sin \; 2m$

$$=(E_1 + 2E_2 \cos m)\sin m \qquad \ldots(7.4)$$

It is interesting to note that the coefficient of sin$m$ is a **variable** and that most of the traditional Indian texts indeed have taken the coefficient of sin$m$ i.e. the *mandaparidhi* p as variable.

Here, it is important to note that in the preceding modern expressions for the equation of the centre, the anomaly m is the angular distance of a mean body from its *perigee*.

i.e. anomaly = mean planet *perigee*

In the Indian classical texts, the *manda* anomaly of a body is its angular distance from its apogee (*mandocca*). For example, the *SS* defines:

Mandakendra (anomaly)*MK* = *Mandocca* mean planet.

Therefore to suit the Indian classical procedure we put m = 180° *MK* in (7.4). We have

$$MPH = \left(E_1 + 2E_2 \cos\left(180^o - MK\right)\right)\sin\left(180^o - MK\right) \qquad \ldots(7.5)$$
$$= \left(E_1 - 2E_2 \cos\left(MK\right)\right)\sin\left(MK\right)$$

For $MK = 0°$, $90°$ and $180°$ the coefficient of $\sin m$ becomes E_1 $62E_2$, E_1 and E_1+2E_2 respectively. These corresponding to *manda paridhi* for the *manda* anomaly of 0°, 90° and 180°. We denote these respectively by p_1, p_2 and p_3.

Table 7.1: The manda peripheries taking two terms

Planet	P_1 (mk=0°)	P_2 (mk=90°)	P_1 (mk=180°)
Kuja	59.42089	67.26894	75.11699
Budha	109.8149	147.2895	184.7642
Guru	32.6452	34.73987	36.83454
Śukra	4.800456	4.841145	4.881833
Śukra	36.10916	38.70979	41.31033

Example: For Mars (Kuja) the values of the equation of the centre according to our revised *paridhi* for a few sample values of the *manda* anomaly are listed in Table 7.2

Table 7.2: Kuja's Mandaphala with two terms

MK	*Paridhi*	*MPH* (Revised)	*MPH* (Modern)
0°	59.42089	0	0
30°	60.47233	289.0533	291.5343
60°	63.34492	525.8681	524.0782
90°	67.26894	646.121	639.3685
120°	71.19296	591.6337	588.4923
150°	74.06555	354.2373	356.9262
180°	75.11699	0	0

Now, by considering the equation of the centre of Mars for the *manda* anomaly from 0° to 180° at intervals of 30° we notice that the error is less than about 3.1 arc minutes in most cases except for *MK*=90° where it is 6.75 arc minutes.

In the case of Mercury (Budha) the deviation of the values according to the revised *mandaphala* from those of modern computation is noticeable. In fact it is as much as about 30.099 arc minutes. This pronounced gap is caused by the high eccentricity of Mercury's orbit.

8. THE ANOMALOUS CASE OF MERCURY (BUDHA)

In the case of Mercury there is always a marked deviation in the traditionally calculated position from the observed or modern calculated position of Mercury. This is mainly due to the fact that the classical Indian astronomers did not adequately account for the high eccentricity of the planet's orbit.

The *manda* periphery, as prescribed by *SS*, varies from 28° to 38° for Budha. But, actually, considering the eccentricity of the planet's orbit its *manda* periphery should vary from about 109°.8 to about 184°.7. The coefficient in the *manda* equation depends on the eccentricity e of the planet's orbit. In fact the coefficient to a first approximation is given by $2e$. In the case of Budha $e = 0.205656$ currently and hence the co-efficient of the *manda* equation turns out to be about 23°.566.

Considering the first two terms, equation (7.4) yields the values of the Budha's *mandaphala* for $MK = 0°$ to 180°, interval of 30°

Table 8.1: Budha's *Mandaphala* with two terms

MK	*Paridhi*	*MPH* (Revised)	*MPH* (Modern)
0°	109.8149	0	0
30°	114.8355	548.2991	579.8515
60°	128.5522	1063.118	1067.58
90°	147.2895	1406.511	1376.412
120°	166.0268	1373.03	1366.065
150°	179.7435	858.2118	896.1924
180°	184.7642	0	0

In order to get improved values for the *mandaphala* of Budha in particular we will have to consider additional terms in the modern sine series expansion for the equation of the centre.

Table 8.2: Kuja's *Mandaphala* with five terms

MK	*Paridhi*	*MPH* (arcsine)	*MPH* (Revised)	*MPH* (Modern)
0°	60.26951	0	0	0
30°	61.06177	291.8775	291.5485	291.5343
60°	63.37153	526.0908	524.0784	524.0782
90°	66.94862	643.0076	639.3121	639.3685
120°	71.16046	591.3609	588.4921	588.4923
150°	74.75739	357.5581	356.9401	356.9296
180°	76.20208	0	0	0

For example, the results for Mars (Kuja) and Mercury (Budha) are shown in Table (8.2) and Table (8.3) for which we have considered the first five terms in the expansion.

Table 8.3 Budha's *Mandaphala* with five terms

MK	*Paridhi*	*MPH* (arcsine)	*MPH* (Revised)	*MPH* (Modern)
0°	117.7177	0	0	0
30°	120.548	575.5744	574.8515	578.2549
60°	129.0918	1067.581	1067.58	1085.447
90°	143.8349	1372.522	1376.412	1456.35
120°	165.1846	1366.065	1366.065	1404.731
150°	187.8495	896.9149	896.1924	907.3468
180°	198.1944	0	0	0

Note: In the expression for the *mandaphala*, by approximating $\sin E \sim E$ we get the values as tabulated in column IV. On the other hand, if arcsine is considered then the corresponding *mandaphala* values are tabulated in column III

9. NĪLAKAṆṬHA'S VALUES FOR THE *MANDA* PERIPHERIES

It is interesting that the famous Kerala astronomer Nīlakaṇṭha Somayājī has revised the *manda* peripheries of the five planets. The values according to his

Siddhāntadarpaṇa (*SD*) are compared with the traditional values according to the *Āryabhaṭīyam* and the *Sūryasiddhānta* in Table 9.1

From Table 9.1, we find that (i) Nīlakaṇṭha's *SD* takes constant values for the *manda* peripheries of all the bodies whereas according to the other texts, these are variable for the five planets; (ii) Āryabhaṭa has assigned constant peripheries for the Sun and the Moon; (iii) Nīlakaṇṭha's value for Mercury's (Budha) *manda* periphery viz., 63° is a substantial departure from the traditional value of 28° 6 30° (according to *SS*); and (iv) Our revised *manda* peripheries yield the *mandaphalas* of the bodies comparable to those of modern computations. It is particularly to be noted that our revised periphery for Mercury viz. 117°.72-198°.19 is markedly far higher than the traditional value.

Table 9.1 : The *manda* peripheries according to three texts

Planet	*Āryabhaṭīyam*	*Sūryasiddhānta*	*Siddhāntadarpaṇa*	Modern Proposed
Ravi	13°30'	13°40' - 14°	13°30'	11°.78 - 12.29°
Chandra	31°30'	31°40' - 32°	31°30'	36°.98 - 42°.43
Kuja	63° - 81°	63° - 81°	72°	60°.27 - 76°.20
Budha	22.5° - 31.5°	22.5° - 31.5°	63°	117°.72 - 198°.19
Guru	31.5° - 36.5°	31.5° - 36.5°	36°	32.77° - 36°.97
Śuka	9° - 18°	9° - 18°	13.30°	4°.80 - 4°.88
Śani	40.5° - 58.5°	40.5° - 58.5°	45°	36°.28 - 41°.52

10. CONCLUSION

In the preceding pages we have analyzed the procedure for the *mandaphala* (equation of the centre) of the five planets according to *MKS* and have worked out the hitherto unknown procedure for the same. Further, in keeping with the socio-religious need to revise the Indian calendrical system, we have suggested revised *manda* peripheries of the Sun, the Moon and the planets. We have obtained the critical *manda* anomaly and corresponding maximum *mandaphala* of the heavenly bodies according to *MKS* as well as our proposed *manda paridhis*. We are working on a similar analysis for the *śīghra* equation (the equation of the conjunction) and propose to publish the same.

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